Time Domain Transfer Function of the Induction Motor

N N Barsoum

Electrical Engineering Program, School of Engineering and Information Technology, University Malaysia Sabah
nader@ums.edu.my

Abstract: This paper discusses with figures and presents the differences observed between Laplace transform and Fourier transform in time domain response, when transforming the transfer function from the frequency domain of the perturbed shaft induction motor under hunting condition, related to applied small signal stability performance

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INTRODUCTION

The remark about the magnitude of the transfer function in [1,4] and its characteristics with the perturbation frequemcy β in frequency domain, gives a relation to the stability characteristics in terms of torque coefficients, T_D and T_{St}, in time domain. It shows the time response of the perturbation velocity pΔθ and the relation to stability and instability of both real and complex roots [1]. The response of pΔθ is observed in this paper for the induction motor in synchronous flux wave reference frame [3]. Two types of transformations are used in this paper, Laplace and Fourier.

These will show the similarity in the response of pΔθ of the motor in case of stable machine, and the unsimilarity for the unstable machine [1,2]; where Laplace is available to apply on a single operating condition, given by the eigenvalue, while Fourier is for evaluating the response of the transfer function in frequency domain, by considering all the values of β from 0 to ∞, for the same operating condition [5]. This matter is discussed in the follwing section.

LAPLACE AND FOURIER TRANSFORMS FOR TORQUE COEFFICIENTS

In this section both transformations are applied to obtain the response of pΔθ for the induction mlotor, defined by h(t) in time domain, for a given unit impulse of torque ΔT_m. The transfer function in frequency domain is therefore:

$$H(j\beta) = \frac{p\Delta\theta}{\Delta T_m}(\beta)$$, which transformed into

$h(t) = p\Delta\theta(t)$, for $\Delta T_m = 1$, in time domain.

In Laplace transform, the solution of h(t) is given by equation (4), which contains all the eigenvalues for particular operating condition. This is solved in the Appendix by means of partial fraction operation (see equation (3)) at $\Delta T_m = 1$. This depends on the numerical solution of the eigenvalue, which are obtained from the characteristic equation, found in the denominator of equation (2).

It can be seen that the solution h(t) is directly related to the condition for stability. It is of damped oscillation for stable machine, increasing with oscillation if the machine has an unstable complex eigenvalue, and increasing exponentially if it has an unstable real eigenvalue [6]; while the retain eigenvalue remain stable. These are shown in Figs(1), but the behaviour is different in case of applying Fourier transform.

Fourier transform has been defined in [6,7] as:

$$H(j\beta) = \int_0^\infty h(t)\, e^{-j\beta t}\, dt$$ and

$$h(t) = \frac{1}{\pi}\int_0^\infty H(j\beta)\, e^{j\beta t}\, d\beta$$

which can be modified by truncating β-range at Ω [6] as follows:

$$h(t) = \frac{1}{\pi}\int_0^\infty H(j\beta)\, e^{j\beta t}\, \frac{\sin(\frac{\beta\pi}{\Omega})}{(\frac{\beta\pi}{\Omega})}\, d\beta \qquad (1)$$

In (1), Ω needs to be sufficiently high $\approx \frac{1}{\Delta t}$, where Δt is the smallest time step of interest, as it discussed in [5,6], so that the integration of H(jβ) needs a sufficiently high t-value, for converging $h(t)\rightarrow 0$. The way of doing this is to get the Gibbs oscillation problem [5] well away from the main swing frequency, as it appears in Fig(3A).

The transformation equation (1) has been established in [6] for all $0 \le \beta \le \Omega$ and used in this paper to apply on:

$$H(j\beta)=\frac{\beta}{T_{St}(\beta)+j\beta T_D(\beta)}=\frac{\beta(T_{St}-j\beta T_D)}{T_{St}^2(\beta)+\beta^2 T_D^2(\beta)}$$

in real and imaginary parts, or:

$$H(j\beta)=\frac{\beta}{\sqrt{T_{St}^2(\beta)+\beta^2 T_D^2(\beta)}}\Bigg|\underline{\tan^{-1}-(\frac{\beta T_D}{T_{St}})}$$

in magnitude and angle, for induction motor.

It can be appreciated from the characteristics of the transfer function TF = |H(jβ)| in [1,2,4], that if the machine is stable, then Fourier h(t) is similar to Laplace h(t) when the dominant eigenvalue is lightly damped. (It should be noticed that Laplace transform is obtained by the eigenvalue, i.e. for a single value of β, which is the frequency of oscilation γ of the dominant root; while Fourier is concerned with all β-values between 0 and Ω.) If the machine works on the stability boundary, where the real part of the dominant root σ = 0, then TF = ∞ at β = γ of the same root of having σ = 0. Thus, Laplace h(t) will oscillate freely at the stability boundary of the complex root, but it is constant at the boundary of the real root (where γ = 0). While Fourier h(t) does not exist in both cases, since |H(jβ)| = ∞ at β = γ (if complex) or at β = 0 (if real). Similarly, for a machine with heavily damped pair of eigenvalue, that |H(jβ)|→0 as σ→∞ and therefore it contributes very little to H and consequently Fourier h(t)→0. Note that, |H(jβ)|→0 usually occurs for induction motor when β→∞, so that h(t) of (1) → 0, but starts to oscillate at high t-values, only in case of having very high Ω-value, and couses Gibbs problem. These purposes are all illustrated in Figs(1,2,3).

If the machine is unstable, whether dynamically (in complex root) or statically (in real root) [1] or both together, then according to the existence theorems Fourier h(t) excludes all the unstable eigenvalue and measures T_D and T_{St} as the coefficients of the sum of the remaining stable roots. Obviously, it is true for Fourier transform in the high-order systems, which shows the response for the stable roots only, without measuring what is included on the right hand side of complex plane within the coefficients of the transfer function, as the existence theorems state. Therefore, Fourier h(t) concerns only with the stable roots, but if one root is on the boundary then H(jβ) does not exist. Figs(2) show the differences of this method corresponding to Figs(1).

However, this seems unlikely to happen in the second-order system with a pair of unstable roots. The remaining stable roots are none, and so T_{St} and T_D should be in such a way to give H(jβ) = 0 for all β. This seems to require $|T_{St}+j\beta T_D| = \infty$ for all β, which seems impossible.

In Figs(2,3B,3C), Ω is chosen to be 4 per unit (PU), and the step of β is 0.001 PU. So the integration (1) is of time step equals 0.25 sec. and includes 4000 value for each T_{St} and T_D. However, in Fig(3A), Ω = 10 PU, which is the reason of finding Gibbs oscillation at high t as shown while it is not found in the other figures (3A,3B) at the same t = 1000 sec.

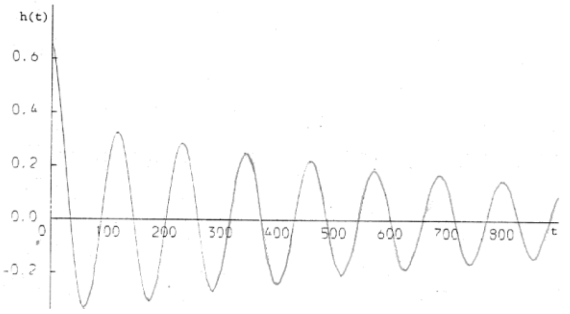
Figs(2) show the digital, discrete Fourier transform (DFT, in which the integration (1) is effectively a summation of H(jβ) with an infinitizmal steps in a long range of β [6, 7]) of h(t) and explain how the characteristics of the coefficients T_D and T_{St} diagnose the stability characteristics in very different way than the characteristics of Figs(1), which are investigated by Laplace transform. This difference appears in h(t) response, when it carries the effects of the harmonics (in amplitude and frequency) in some steps of β if it is obtained by Fourier equation (1), while these harmonics are not involved if it is obtained by Laplace equation (4).

Harmonic effect appears as some ripples occur with the fundamental wave, and can be seen from the cases of heavily damped and real root on the boundary Figs(2C,2D). In all cases, the magnitudes and frequencies are not the same, comparing Laplace (fundamental) with Fourier (fundamental plus harmonics), but Fourier also shows the cases of stability, instability and boundary of each eigenvalue in different ways of Laplace. These are shown in Figs(1,2) and illustrate that Fourier h(t) can define all these conditions, but does not recognise the instability in h(t) according to an unstable root. This in fact indicates that Fourier is only concerning with the remaining stable roots in measuring T_{St} and T_D coefficients from β = 0 to Ω, as shown in Figs(2E,2F).

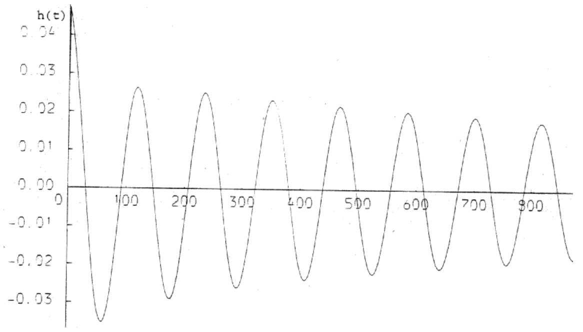
Although Fourier h(t) shows the case of stability boundary in Figs(2C,2D), which are similar to Figs(1C,1D) of Laplace (but are not realy identical), Fourier function in frequency domain H(jβ) at β = γ of the root on the boundary is infinity, and Fourier h(t) of that root does not exist. But in principle the effect of the root on the boundary dominates the solution of h(t), since the other roots are stable and attractive. Thus, the solution is of constant amplitude of the peak values, as shown in the figures.

CONCLUSION

This paper shows the difference between two methods of transformation, Laplace and Fourier on the response of the output speed pΔθ for induction motor. These differences occurred in two cases, one in finding an unstable root, and one in finding a root on the boundary. This indicates that the response of an unstable machine cannot betruly observed by using Fourier transform. It is, therefore, important to put the machine under test for stability, during the practical work of design, using the method of parameter estimation with Fourier transform. This must be applied when the machine variables are expressed in synchronous flux wave reference frame (not the stator reference frame), since the variables are time-invarient.

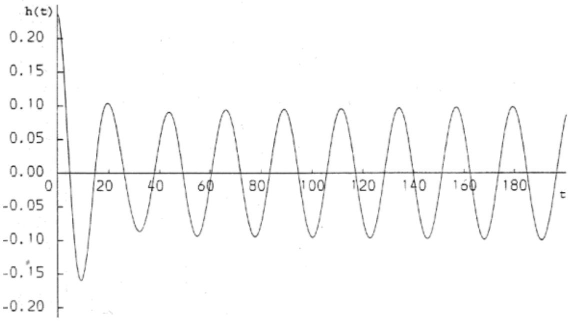


FIG(1A) Laplace transform at unit impuls torque

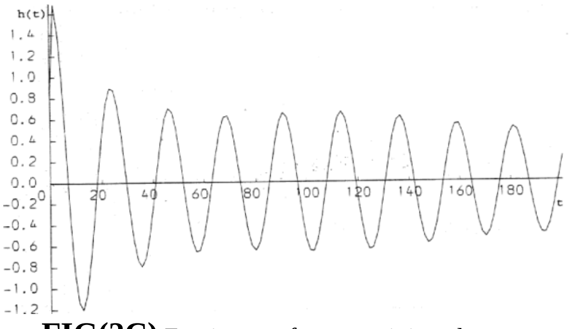


FIG(2A) Fourier transform at unit impuls torque

Induction Motor Parameters in per unit are:
R_s = 0.03, R_r = 0.015, L_{ls} = 0.1, L_{lr} = 0.1, M = 4, S = 0.0, J = 745.0, v = ω = 0.13
Figs.A, are chosen for lightly damping operation.

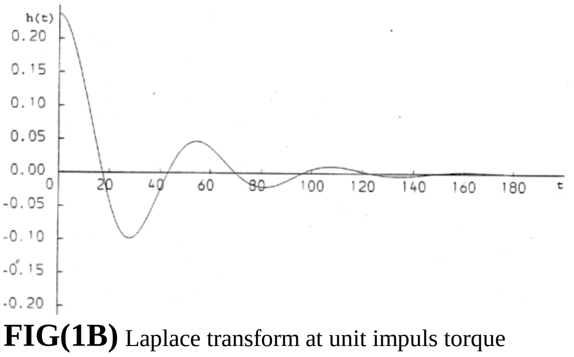


FIG(1C) Laplace transform at unit impuls torque

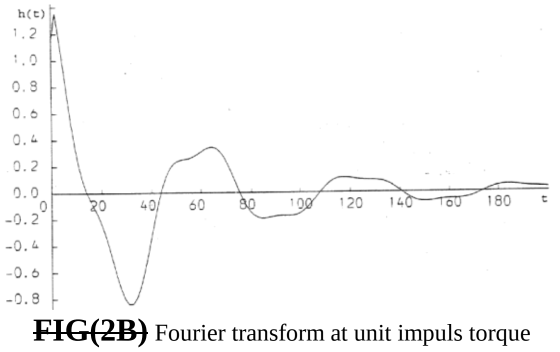


FIG(2C) Fourier transform at unit impuls torque

Induction Motor Parameters in per unit are:
$R_s = R_r$ = 0.01, $L_{ls} = L_{lr}$ = 0.08, M = 3, J = 36
S = 0.0625, v = ω = 0.3
Figs.C, are chosen for complex root on stability boundary.

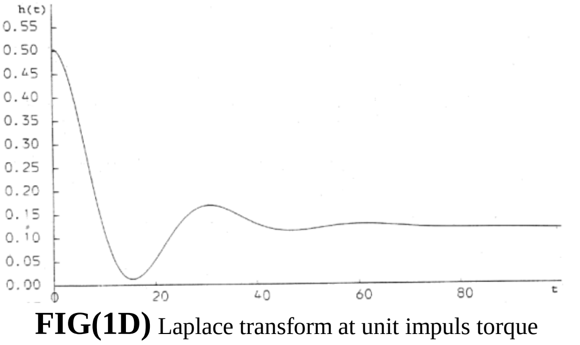


FIG(1B) Laplace transform at unit impuls torque

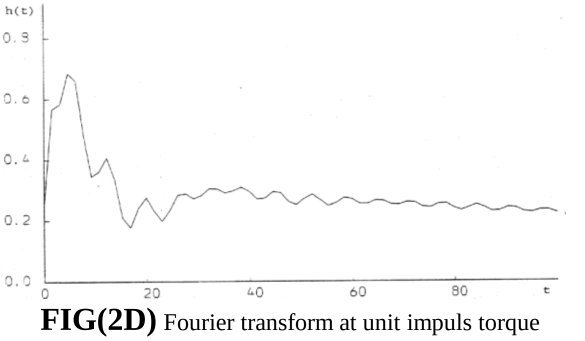


~~FIG(2B)~~ Fourier transform at unit impuls torque

Induction Motor Parameters in per unit are:
$R_s = R_r$ = 0.01, $L_{ls} = L_{lr}$ = 0.08, M = 3, v = 0.09, S = 0.0625, J = 36, ω = 0.3
Figs.B, are chosen for heavily damping operation.

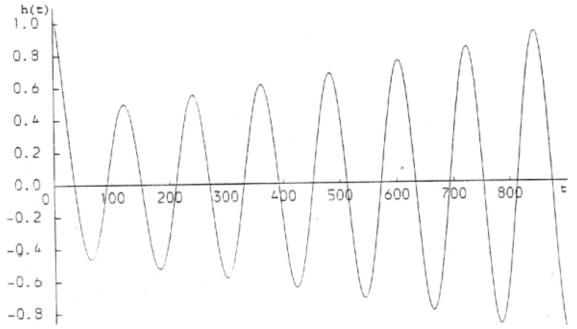


FIG(1D) Laplace transform at unit impuls torque

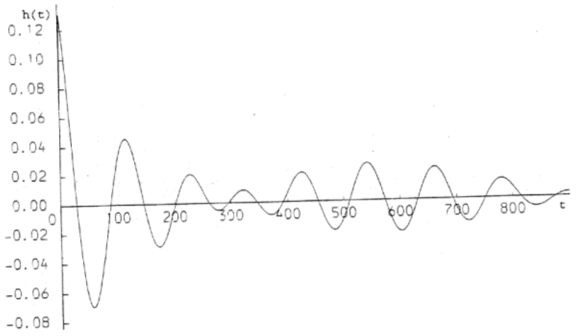


FIG(2D) Fourier transform at unit impuls torque

Induction Motor Parameters in per unit are:
R_s = 0.025, R_r = 0.015, $L_{ls} = L_{lr}$ = 0.1, M = 3.5
S = 0.075, J = 62.8, v = ω = 1.0
Figs.D, are chosen for real root on stability boundary.

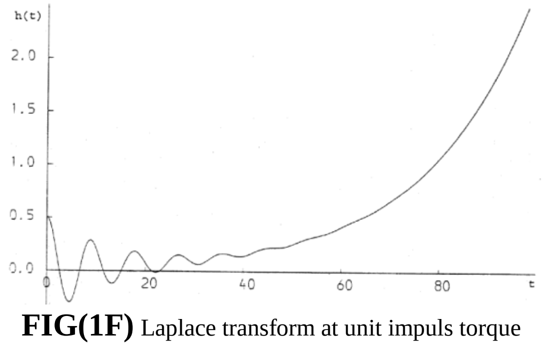


FIG(1E) Laplace transform at unit impuls torque

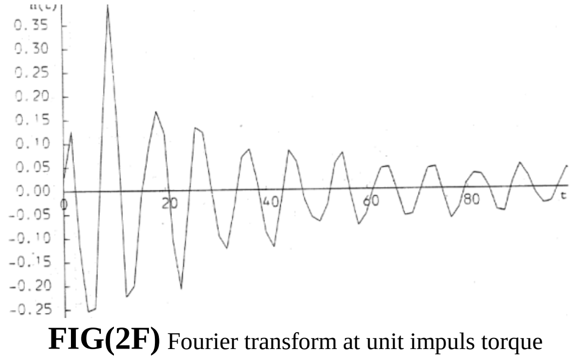


FIG(2E) Fourier transform at unit impuls torque

Induction Motor Parameters in per unit are:
R_s = 0.03, R_r = 0.02, L_{ls} = L_{lr} = 0.1, M = 5
S = 0.005, J = 314, ω = 0.1, v = 0.08
Figs.E, are chosen for an unstable complex root.

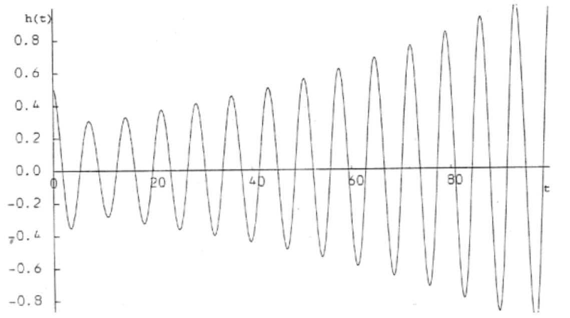


FIG(1F) Laplace transform at unit impuls torque

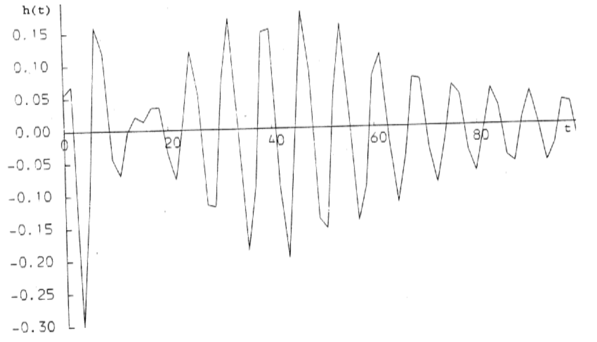


FIG(2F) Fourier transform at unit impuls torque

Induction Motor Parameters in per unit are:
R_s = 0.025, R_r = 0.015, L_{ls} = L_{lr} = 0.1, M = 3.5
S = 0.075, J = 62.8, ω = 1.3, v = 5.6
Figs.F, are chosen for an unstable real root.

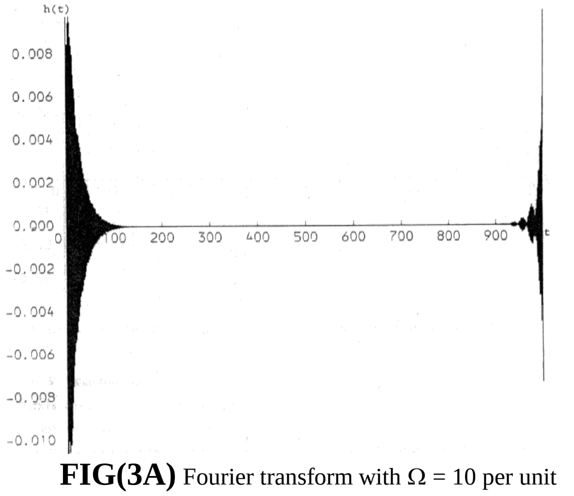


FIG(1G) Laplace transform at unit impuls torque

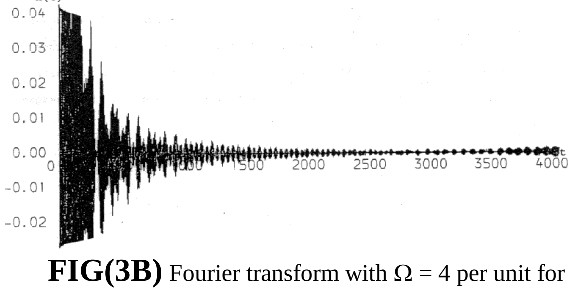


FIG(2G) Fourier transform at unit impuls torque

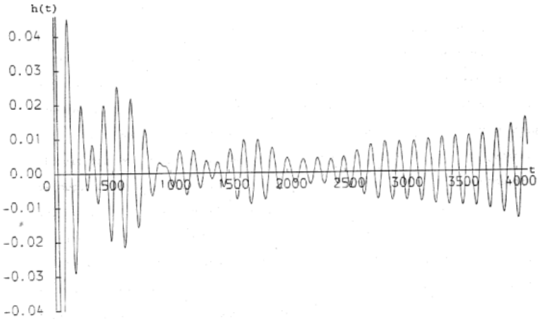
Induction Motor Parameters in per unit are:
R_s = 0.035, R_r = 0.015, L_{ls} = L_{lr} = 0.1, M = 3.5
S = 0.085, J = 62.8, ω = 1.0, v = 5.6
Figs.G. are chosen for both real and complex roots are unstable.



FIG(3A) Fourier transform with Ω = 10 per unit for stable machine



FIG(3B) Fourier transform with Ω = 4 per unit for the same case of Fig(3A)



FIG(3C) Fourier transform with Ω = 4 for unstable machine, in complex root

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APPENDIX

The solution of the output perturbation velocity pΔθ in the dynamic machine equation [2,3] for the induction motor is h(t), given by Laplace transform where h(t) is the response of pΔθ in time-domain to a unit impulse of the torque input ΔT_m.

By applying Laplace transform to the dynamic machine equation [3] with $\Delta T_m(\theta) = 1$ and all the initial values of the perturbation currents and speed are zero which can be deduced from the motional impedance matrix at t = 0 where, p = 0 and ω = 0. Therefore, the Laplace transform of pΔθ is denoted by pΔθ(ρ), where:

$$p\Delta\theta(\rho) = \frac{B_5\rho^4 + B_4\rho^3 + B_3\rho^2 + B_2\rho + B_1}{B_5\rho^5 + B_4\rho^4 + (B_3 + C_3)\rho^3 + (B_2 + C_2)\rho^2 + (B_1 + C_1)\rho + C_0} \quad \text{............ (1)}$$

where ρ is the Laplace symbol, and the coefficients B_1 to B_5 and C_0 to C_3 are defined in [2,3] in terms of machine parameters.

In particular operating condition, the expresion of pΔθ (ρ) can be written as follows:

$$p\Delta\theta(\rho) = \frac{B_5\rho^4 + B_4\rho^3 + B_3\rho^2 + B_2\rho + B_1}{(\rho - \sigma_1 - j\gamma_1)(\rho - \sigma_1 + j\gamma_1)(\rho - \sigma_2 - j\gamma_2)(\rho - \sigma_2 + j\gamma_2)(\rho - \sigma_3)} \quad \text{........... (2)}$$

where, $(\sigma_1 \pm j\gamma_1)$, $(\sigma_2 \pm j\gamma_2)$ and σ_3 are the eigenvalue of the induction motor model [2] at that operating point.

Equation (2) can be splitted into 5-terms by the method of the partial fraction as follows:

$$p\Delta\theta(\rho) = \frac{K_1}{\rho - \sigma_1 - j\gamma_1} + \frac{K_1^*}{\rho - \sigma_1 + j\gamma_1} + \frac{K_2}{\rho - \sigma_2 - j\gamma_2} + \frac{K_2^*}{\rho - \sigma_2 + j\gamma_2} + \frac{K_3}{\rho - \sigma_3} \quad \text{...........(3)}$$

where:

$K_1 = X_1 + jY_1$ $\quad K_2 = X_2 + jY_2$

$K_1^* = X_1 - jY_1$ $\quad K_2^* = X_2 - jY_2$

(* denotes the conjugate), and equation (3) can, therefore, be written as:

$$p\Delta\theta(\rho) = \frac{X_1(\rho - \sigma_1) - Y_1\gamma_1}{(\rho - \sigma_1)^2 + \gamma_1^2} + \frac{X_2(\rho - \sigma_2) - Y_2\gamma_2}{(\rho - \sigma_2)^2 + \gamma_2^2} + \frac{K_3}{\rho - \sigma_3}$$

where:

$$X_1 = \frac{ax_1 + by_1}{x_1^2 + y_1^2} \qquad X_2 = \frac{cx_2 + dy_2}{x_2^2 + y_2^2}$$

$$Y_1 = \frac{bx_1 - ay_1}{x_1^2 + y_1^2} \qquad Y_2 = \frac{dx_2 - cy_2}{x_2^2 + y_2^2}$$

$$K_3 = \frac{B_5\sigma_3^4 + B_4\sigma_3^3 + B_3\sigma_3^3 + B_2\sigma_3 + B_1}{[[(\sigma_3 - \sigma_1)^2 + \gamma_1^2]][(\sigma_3 - \sigma_2)^2 + \gamma_2^2]]}$$

$$a = B_5(\sigma_1^4 - 6\sigma_1^2\gamma_1^2 + \gamma_1^4) + B_4\sigma_1(\sigma_1^2 - 3\gamma_1^2) + B_3(\sigma_1^2 - \gamma_1^2) + B_2\sigma_1 + B_1$$

$$b = 4B_5\sigma_1\gamma_1(\sigma_1^2 - \gamma_1^2) + B_4\gamma_1(3\sigma_1^2 - \gamma_1^2) + 2B_3\sigma_1\gamma_1 + B_2\gamma_1$$

$$c = B_5(\sigma_2^4 - 6\sigma_2^2\gamma_2^2 + \gamma_2^4) + B_4\sigma_2(\sigma_2^2 - 3\gamma_2^2) + B_3(\sigma_2^2 - \gamma_2^2) + B_2\sigma_2 + B_1$$

$$d = 4B_5\sigma_2\gamma_2(\sigma_2^2 - \gamma_2^2) + B_4\gamma_2(3\sigma_2^2 - \gamma_2^2) + 2B_3\sigma_2\gamma_2 + B_2\gamma_2$$

$$x_1 = -\gamma_1^2[2(\sigma_1 - \sigma_2)(\sigma_1 - \sigma_3) + (\sigma_1 - \sigma_2)^2 - (\gamma_1^2 - \gamma_2^2)]$$

$$y_1 = \gamma_1[(\sigma_1 - \sigma_3)\{(\sigma_1 - \sigma_2)^2 - (\gamma_1^2 - \gamma_2^2)\} - 2\gamma_1^2(\sigma_1 - \sigma_2)]$$

$$x_2 = -\gamma_2^2[2(\sigma_2 - \sigma_1)(\sigma_2 - \sigma_3) + (\sigma_2 - \sigma_1)^2 - (\gamma_2^2 - \gamma_1^2)]$$
$$y_2 = \gamma_2[(\sigma_2 - \sigma_3)\{(\sigma_2 - \sigma_1)^2 - (\gamma_2^2 - \gamma_1^2)\} - 2\gamma_2^2(\sigma_2 - \sigma_1)]$$

Thus, the solution of pΔθ in time-domain is the Laplace inverse of pΔθ(ρ) of equation (3), which is equal to h(t), where:

$$\begin{aligned} h(t) = K_3 e^{\sigma_3 t} + e^{\sigma_1 t}(X_1 \cos\gamma_1 t - Y_1 \sin\gamma_1 t) \\ + e^{\sigma_2 t}(X_2 \cos\gamma_2 t - Y_2 \sin\gamma_2 t) \end{aligned} \quad ....(4)$$